CGH Imager and Holographic Image: Number of Pixels and Resolution

We demonstrated our CGH (Computer Generated Hologram) game demo at Photonics West 2019 using a phase microdisplay. People liked it, and we heard lots of interesting questions about it.

We realized there is a big confusion regarding CGH. People asked: What is the resolution of the far-field image? And: What if you have smaller pixels on the CGH?

Here we're trying to explain more about these CGH parameters.

First, we should understand difference between resolution on the microdisplay (also known as Spatial Light Modulator, SLM, imager, or LCoS) , and holographic image resolution in the far-field light diffracted off the SLM. It's different and depends on imager size, distance, and so on.

We decided to put examples first, and math later, so you don't need to scroll down to get the key ideas.

Following examples don't consider coherency properties of laser beam. Rigorous treatment without this approximation makes math much more complicated. But the approximation can be used to understand inherent properties of CGH images.

Examples

Projection

1. Using green laser ($\lambda = 532$ nm) and full-HD imager (number of pixels $M = 1920$) with pixel size $d = 4.0$ µm, imager size $H = 1920 \times 4.0 = 7.68$ mm, and we can create image with field-of-view $\varphi = 7.62°$ by equation (B) below. If image is projected on screen located at $z = 10"$ (254 mm) from CGH, image size $D = 33.8$ mm by simple trigonometry. The image pixel size Δ (which is the minimum resolvable feature at the image plane) is approximately 17.6 µm by equation (A) below. So the far-field holographic image can resolve approximately 33,800 / 17.6 ≈ 1920 pixels, matching the imager's resolution, as expected. Far-field image has been formed without imaging lens, which is very cool, and retains full resolution of imager.

Near-eye

2. Same imager used in near-eye AR glasses at same wavelength still creates 7.62° FOV, which is quite small. And angular resolution is approximately 7.62 / 1920 ≈ 0.238 arc minutes. This is well below human eye visual resolving capability, which is approximately 1 arc minutes. So in this case small (angularly) image is over-resolved by factor of more than 4 times. Not a good match.

3. What if we use same wavelength with again full-HD imager, but now double the pixel size to 8 µm? Imager size equals 15.4 mm. FOV equals 3.81°. Too small. And angular resolution approximately 0.119'. Way too fine for human eye! This is wrong direction.

4. What if we use same wavelength and full-HD with 1.0 µm pixels? Imager size equals 1.92 mm. FOV equals 30.5°. Quite good for one eye. And angular resolution approximately 0.95 arc minutes. Good match to eye. But (see below) eye-box too small (window violation).

clarify.one
hi tech :) job done right

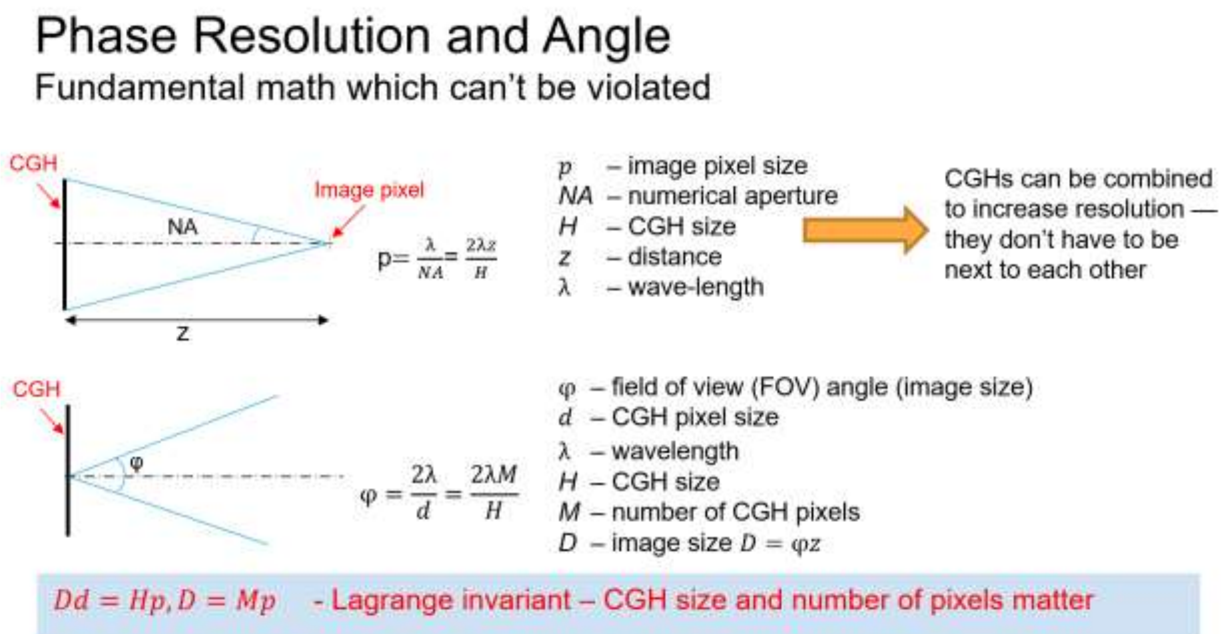
5. Finally, what if we use same wavelength and 4k (3840 pixels) with 0.5 µm pixels? Imager size equals 1.92 mm, FOV equals 61.0°, angular resolution approximately 0.95 arc minutes. Excellent. But 4k with ½ µm pixels is well beyond state-of-art for imagers. And still a window violation.

Beam steering

6. In such applications we can get away with relatively low-resolution imager, since we don't need to create image with lots of details, just a confining spot. So even SVGA 800 × 600 imager with 8 µm pixels is sufficient. Let's calculate. Imager size H = 6.4 mm. If target at z = 254 mm then angular resolution ≈ 0.29' and spot diameter ≈ 21.1 µm. Comfortable size for biological optical tweezers for example.

Equations

Here is full (but still incoherent) math…



Let's say we create holographic image on a screen located at some finite distance z from CGH imager. In such case the imager is the part of optical system for which exit pupil coincides with the imager. For simplicity, we neglect laser beam coherency and use conventional formulas used in "traditional" non-coherent optics. Since imager has rectangular shape, we must treat each direction separately.

One-dimensional resolution criterion, which is similar to Rayleigh resolution criterion:

$$\Delta = \frac{\lambda}{2NA} = \frac{\lambda z}{H} \qquad \text{(A)}$$

where:

Δ = resolution at the holographic image plane

λ = wavelength

z = distance between CGH and image plane

H = CGH size

$NA = \frac{\mathrm{H}}{2z}$ = Numerical Aperture (rectangular image, so NA and resolution are different in each direction)

The relationship between field of view (holographic image size) φ and pixel size d is governed by the diffraction equation:

$$\varphi = \frac{2\lambda}{d} = \frac{2\lambda M}{H} \tag{B}$$

where:

φ = full field of view angle, not half-angle (in radians)

d = pixel size

M = number of pixels (in one dimension)

From these two formulas above we can derive:

$$D = \varphi z = 2M\Delta = 2\frac{H}{d}\Delta \tag{C}$$

where:

D = image size

One can ask where the 2 came from in equation (C). It appears that we double amount of information stored in generated CGH. This is easy to understand from definition of resolution criterion: resolution limit corresponds to shift between points when central maximum of first point coincides with first dark Airy ring of second point. This shift approximately equals half of point size.

If we use pixels size at the holographic image plane, then Equation (C) is nothing but re-written Lagrange invariant:

$$Dd = H\mathrm{p} \tag{D}$$

where:

$p = 2\Delta$ = image pixel size

These equations allow to understand better how LCoS parameters affect holographic image. If we use LCoS with smaller pixels, then LCoS size will be smaller, but image size will actually be larger, so resolution decreases (Δ will be larger). Therefore, it's necessary to increase number of pixels to maintain resolution. Phase likes small pixels for large FOV, and many pixels for high resolution.

Eye box / Window violation

Another important consideration for near-eye displays is eye-box, i.e., area within which observer can move eye and still see the image. Holographers call this the “window”. From optical point of view, imager works as a vignetting aperture, limiting the field of view. Therefore, small imagers limit eye's field of view and limit possibility of eye-shifts, unless imager is used with additional magnifying optics. It is outside the scope of this short note to discuss solutions for this in detail (hint: consider using flat/conformal holographic optics). Please contact us if you want to know more. We’d be glad to help!